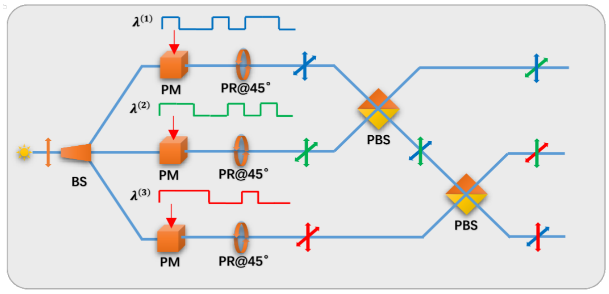

GHZ experiment [Fu et al., arxiv 1609.08904]

Exercise 8.1: GHZ state (3P)

The GHZ state for a three-qubit system with $\mathcal{H} = (\mathbb{C}^2)^{\otimes 3} = \mathbb{C}^8$ is defined by

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big).$$

(a) Compute the reduced 2×2 density matrix of the third qubit and find out whether this qubit is entangled with the other two qubits. (1P)

(b) Calculate the reduced 4×4 density matrix of the first two qubits and calculate the corresponding von-Neumann entropy. (1P)

(c) Is the state determined in (b) separable or entangled? (1P)

Exercise 8.2: Creatioin of entanglement by measurement (4P)

Consider the pure 4-qubit state

$$|\Psi_\alpha\rangle = |\psi_\alpha\rangle \otimes |\psi_\alpha\rangle \qquad \text{with} \quad |\psi_\alpha\rangle = \alpha|00\rangle + \sqrt{1-\alpha^2}|11\rangle.$$

Suppose that we perform a projective measurement with the measurement operator

$$\mathbf{M} \;=\; \sigma^z \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} + \mathbb{1} \otimes \mathbb{1} \otimes \sigma^z \otimes \mathbb{1}.$$

The purpose of this exercise is to show that measurements do not always destroy entanglement (converting it into classical correlations of outcomes), but measurements can also be used to create entanglement.

(a) Compute the eigenvalues of $\mathbf{M}$ and their degeneracies. (1P)

(b) Determine the projection operators of the measurement. (1P)

(c) In which state is the system after the measurement given that the measurement result is known? (1P)

(d) What is the probability to obtain a fully entangled state between the two left and the two right qubits? (1P)

Please turn over ⇒

Exercise 8.3: Schmidt decomposition **(5P)**

Determine the Schmidt decomposition of the following 2-qubit states (please sketch your calculation step by step):

(a) $|\psi\rangle = \frac{1}{2}\Big(|00\rangle + |01\rangle + |10\rangle + |11\rangle\Big)$. (1P)

(b) $|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$. (1P)

(c) $|\psi\rangle = \frac{1}{2}\Big(|00\rangle + |01\rangle + 2|10\rangle\Big)$. (3P)

(Σ = 12P)

To be handed in on Monday, December 18, at the beginning of the tutorial.