Quantum Information Theory

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Entropy for gamers [Wikimedia]

Exercise 1.1: Deformed entropy measures (4P)

(a) Prove the the Tsallis entropy $S_q^T = (q-1)^{-1}(1-\sum_i p_i^q)$ (see lecture notes) reduces to the usual Shannon entropy S in the limit $q \to 1$. (1P)

(b) Let A and B two uncorrelated systems, i.e. $p_{i,j}^{AB} = p_i^A p_j^B$ (cf. lecture notes). Show that Tsallis entropy is non-extensive, i.e., (2P)

$$S_q^T(AB) = S_q^T(A) + S_q^T(B) + (1-q)S_q^T(A)S_q^T(B)$$

(c) Show that the Rényi entropy, in contrast to Tsallis entropy, is always extensive on uncorrelated subsystems, i.e. $S_q^R(AB) = S_q^R(A) + S_q^R(B)$ for all $\alpha > 0$. (1P)

Exercise 1.2: Relative entropy (2P)

Let$\{p_1, \dots, p_N\}$ and $\{q_1, \dots, q_N\}$ be two discrete probability distributions. The Kullback-Leibler divergence (also called relative entropy or Kullback-Leibler distance) between the two distributions is defined as

$$D(p||q) = \sum_i p_i \ln \frac{p_i}{q_i} \,.$$

Note that the Kullback-Leibler divergence is generally non-symmetric. Show that

(a) $D(p||q) \geq 0$. Hint: Use Jensen's inequality. (1P)

(b) $D(p||q) = 0$ if and only if the two distributions coincide. (1P)

(Σ = 6P)

To be handed in on Monday, October 23, at the beginning of the tutorial. Normally we have 12P per sheet. This week we have only 6P.